Unit 4: Geometry and Spatial Sense

Introduction

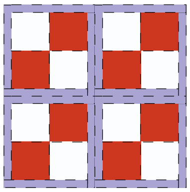
The word geometry means *earth-measuring*. Use of the Pythagorean theorem by the Egyptians to survey land dates back to around 2000 B.C.

In studying geometry, students can quickly see numerous applications for real-life shapes, design, and sculpture. We see parallel lines any time we travel on a highway or park in a marked parking area. Contractors, architects, artists, and engineers use geometry extensively. We see the results of their talents in buildings of every imaginable size and shape. Highways, bridges, and tunnels are also created by people who are experts in geometry.

Contractors, architects, artists, and engineers use geometry extensively.

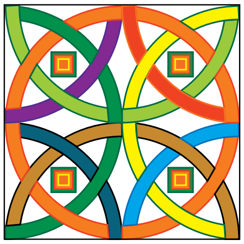
You will learn to identify various types of polygons in this unit. When you look around your home, you will usually find many geometric shapes in upholstery fabrics, curtains, bedspreads, floor coverings, furniture, and artwork.

fabrics



bedspreads

floor coverings



artwork

Lesson One Purpose

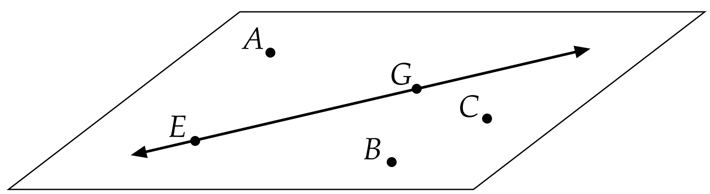
- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)

- Use concrete and graphic models to derive formulas for finding angle measures. (MA.B.1.4.2)
- Understand geometric concepts such as perpendicularity, parallelism, congruency, and similarity. (MA.C.2.4.1)

Geometric Basics

Basic items include **planes**, **points**, and **lines ($\leftrightarrow$)**.

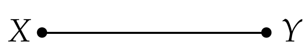
The figure below represents a *plane*—an undefined, two-dimensional geometric surface with no specified boundaries. A plane is a flat surface. We use dots and capital letters to indicate *points*—a location in space that has no **length (*l*)** or **width (*w*)**. See points *A*, *B*, and *C*.



plane

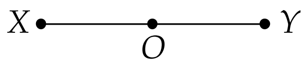
Lines are drawn with arrows at both ends to indicate that the lines are infinitely long. Note how the symbol ($\leftrightarrow$) drawn over two capital letters is used to describe a line. See line *EG* ($\overleftrightarrow{EG}$) above.

See a representation of **line segment (—)** *XY* ($\overline{XY}$) below. $\overline{XY}$ indicates it is a segment with ends at points *X* and *Y*. Note how the symbol (—) drawn over two capital letters is used to describe a line segment. The symbol has no arrows because the line segment has a defined beginning and end called **endpoints**. *Endpoints* are either of the two points marking the end of a line segment. *X* and *Y* are endpoints of the line segment *XY* ($\overline{XY}$).



Line segments have measurable *length*. However, it is impossible to measure the lengths of lines because they extend forever in opposite directions.

Suppose that $\overline{XY}$ below has a point O placed on the segment so that the distance from X to O is the same as the distance from O to Y.



We describe the length of $\overline{XO}$ as 6. We write $\overline{XO} = 6$. Likewise the length of $\overline{OY} = 6$. Since the segments have the same length, we can say that the segments are **congruent** (≅), the same shape and the same size.

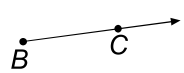
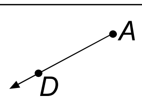
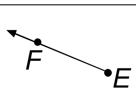
$$\overline{XO} \cong \overline{OY}$$

The symbol ≅ is read: segment XO is *congruent* to segment OY.

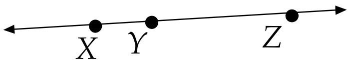
We also draw **rays** (→), a portion of a line that begins at a point and goes on forever in one direction. Rays can be used to construct **angles (∠)**. *Angles* are shapes made by two rays extending from a common endpoint. *Rays* can also be part of lines.

Note how the symbol → drawn over two capital letters is used to describe a ray. Rays are always named with the *endpoint* listed *first* to describe the *originating* (beginning) point of the ray. For example, see the chart below. In ray BC ($\overrightarrow{BC}$), B is the *endpoint* and is listed first.

Name of Rays

Drawing	Name	Endpoint
Ray *BC* 	$\overrightarrow{BC}$	*B*
Ray *AD* 	$\overrightarrow{AD}$	*A*
Ray *EF* 	$\overrightarrow{EF}$	*E*

The line to the right can be labeled many ways:



$\overleftrightarrow{XZ}$ or $\overleftrightarrow{ZX}$
$\overleftrightarrow{XY}$ or $\overleftrightarrow{YX}$
$\overleftrightarrow{YZ}$ or $\overleftrightarrow{ZY}$

The line also contains rays. Some possibilities are as follows:

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ name the same ray.

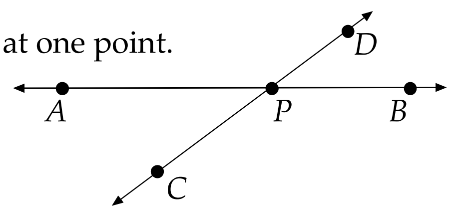
$\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are opposite rays.

$\overrightarrow{XY}$ and $\overrightarrow{YZ}$ are distinctly different rays.

$\overrightarrow{ZY}$ and $\overrightarrow{XY}$ also are different rays.

Note: $\overrightarrow{XZ}$ and $\overrightarrow{ZX}$ are different rays but are not opposite rays because they overlap and do *not* have the same endpoint. $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are opposite rays because they share the same endpoint and do *not* overlap.

When lines **intersect**, they meet or cross at one point.



$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point P.

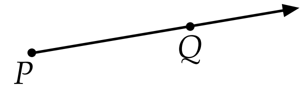
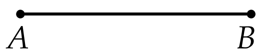
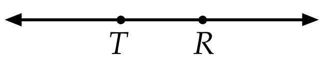
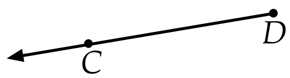
We can also identify segments that lie on $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. For example, $\overline{AP}$ lies on $\overleftrightarrow{AB}$.

The same segment $\overline{AB}$ can be written as $\overline{BA}$. $\overline{CP}$ (or $\overline{PC}$) also lies on $\overleftrightarrow{CD}$.

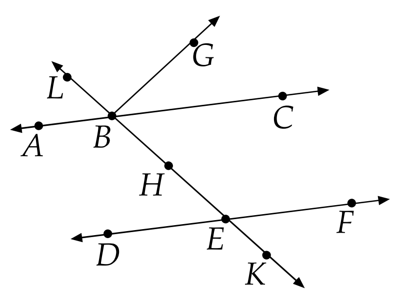
Note: Appendix B contains a list of mathematical symbols and their meanings.

Practice

Use the following to write the **symbol** *and to name the* **endpoint(s)** *of each figure.*

		Symbols	**Endpoint(s)**
1. Ray:		________	________
2. Segment:		________	________
3. Line:		________	________
4. Ray:		________	________

Answer the following using the figure below.



5. Write four names for line *BK*. ______________________

____

6. Name three different line segments that lie on $\overleftrightarrow{BK}$. __________

____

7. Name two lines that intersect. ______________________________

____

8. Name five rays that have the same endpoint. ________________

____

____

9. Are $\overrightarrow{DE}$ and $\overrightarrow{ED}$ the same ray? ____________________________

Explain. ____

____

10. If $\overline{BH}$ and $\overline{HE}$ have the same lengths, are they congruent? ______

____

11. Could $\overleftrightarrow{AC}$ and $\overleftrightarrow{DF}$ be congruent? __________________________

Explain. ____

____

12. Could $\overline{AC}$ and $\overline{DF}$ be congruent? __________________________

Explain. ____

____

13. Name opposite rays that have endpoint *E*. ____________________

____

14. Are $\overrightarrow{DE}$ and $\overrightarrow{FE}$ opposite rays? ____________________________

Explain. ____

____

15. Are $\overline{AB}$ and $\overline{BA}$ the same line segment? ______________________

Explain. ____

____

16. Are $\overrightarrow{AB}$ and $\overrightarrow{BA}$ the same ray? ______________________________

Explain. ____

____

Practice

Use the list below to write the correct term for each definition on the line provided.

angle	**length (*l*)**	**point**
congruent (≅)	**line**	**ray**
endpoint	**line segment**	**width (*w*)**
intersect	**plane**	

________________________ 1. a location in space that has no length or width

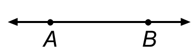
________________________ 2. an undefined, two-dimensional (no depth) geometric surface that has no boundaries specified; a flat surface

________________________ 3. a portion of a line that begins at a point and goes on forever in one direction

________________________ 4. a one-dimensional measure that is the measurable property of line segments

________________________ 5. figures or objects that are the same shape and the same size

________________________ 6. a straight line that is endless in length



________________________ 7. either of two points marking the end of a line segment

________________________ 8. a portion of a line that has a defined beginning and end

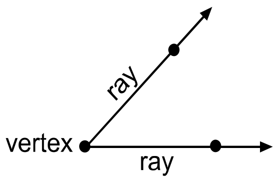
________________________ 9. to meet or cross at one point

__________________________ 10. a one-dimensional measure of something side to side

__________________________ 11. the shape made by two rays extending from a common endpoint

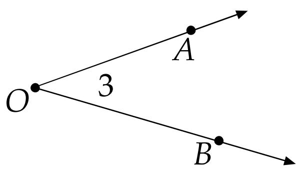
Measuring and Classifying Angles

The *sides* of an angle are formed by two rays (→) extending from a common endpoint called the **vertex**.



Naming an Angle

Consider the following figure.



Remember: The symbol ∠ indicates an angle.

You can name an angle in three ways.

- use a three-letter name in this order: point on one ray; vertex; point on other ray, such as ∠*AOB or* ∠*BOA*
- use a one-letter name: vertex, if there is only one angle with this vertex in the diagram, such as ∠*O*
- use a numerical name if the number is within the rays of the angle, such as ∠3

The angle is formed by rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$. The rays are portions of lines that begin at a point and go on forever in one direction.

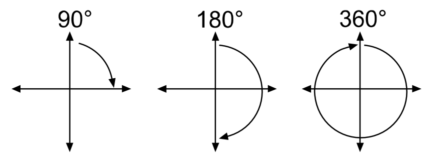
The point *O*, which is the same endpoint for $\overrightarrow{OA}$ and $\overrightarrow{OB}$, is the *vertex* of angle *AOB*. When using three letters to name an angle, the *vertex* letter is listed in the middle.

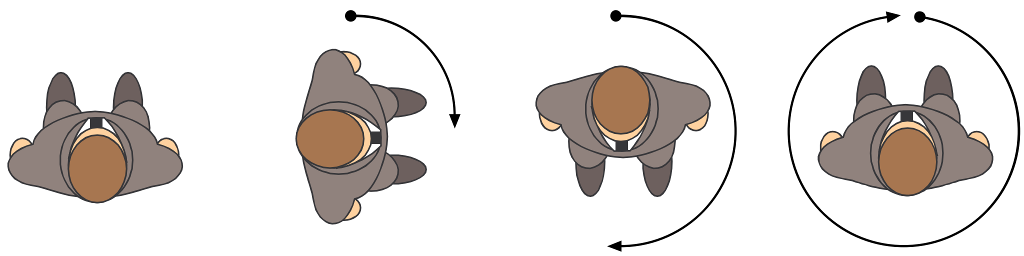
The measure of ∠*O* is written as m∠*O*. Sometimes two (or more) angles have the same measure. When two angles have the same measure, they are congruent.

Measuring an Angle

The **measure (m) of an angle (∠)** is described in **degrees (°)**. When you turn around and face backward, you could say you "did a 180." If you turn all the way around, it is a 360. An angle is a turn around a point. The size of an angle is the measure of how far one side has turned from the other side.

0°	=	no turn
90°	=	right-angle turn
180°	=	straight backward
360°	=	back facing forward again

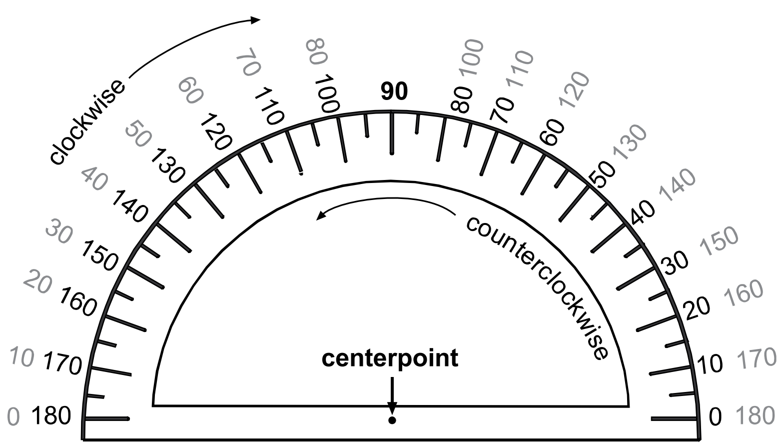




0° = no turn **90° = right-angle turn** **180° = straight backward** **360° = back facing forward again**

Using Protractors to Measure Angles

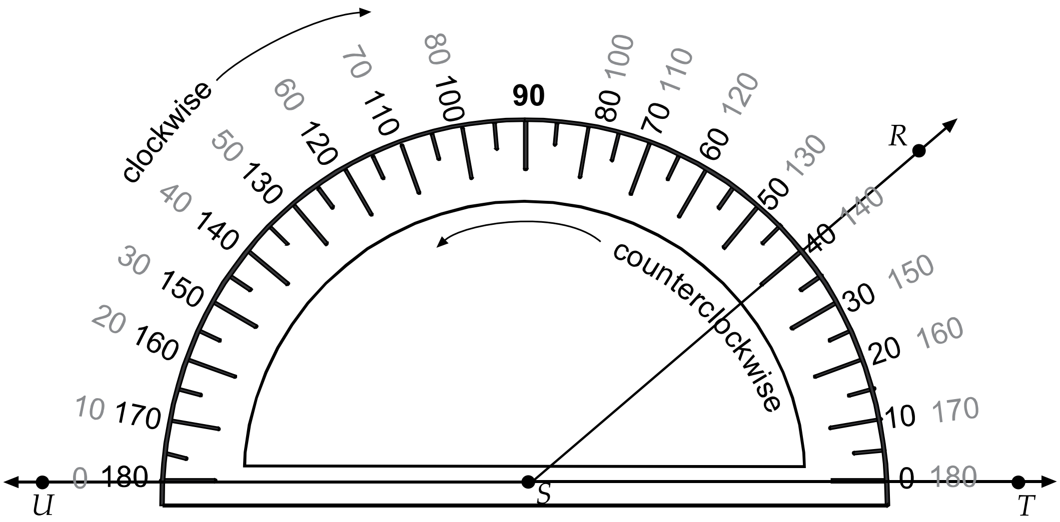
Protractors are marked from 0 to 180 degrees in both a clockwise manner and a counterclockwise manner. We see 10 and 170 in the same position. We see 55 and 125 in the same position. If we estimate the size of the angle before using the protractor, there is no doubt which measure is correct.



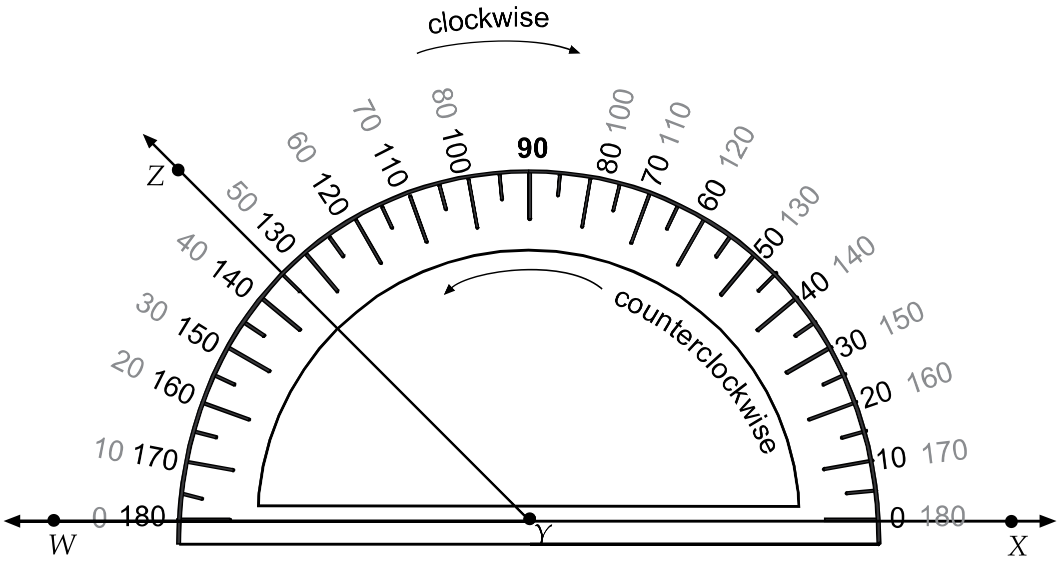
When using a protractor, make sure the vertex is lined up correctly and that one *ray* (⟶) passes through the zero measure. A straightedge is often helpful to extend a ray for easier reading of the measure.

A *protractor* is used to measure angles. Follow these steps to use a protractor.

1. Place the centerpoint of the protractor on the vertex of the angle.
2. Line up the protractor's 0 degree line with one side of the angle.
3. Read the measure of the angle where the other side crosses the protractor.



- The measurement of ∠*TSR* is ___40°___ .
  40° is read 40 degrees.
- The measurement of ∠*USR* is ___140°___ .

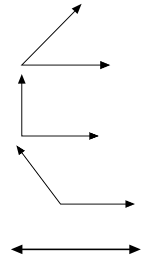


- The measurement of $\angle XYZ$ is ____135°____ .
- The measurement of $\angle WYZ$ is ____45°____ .

Naming Different Size Angles

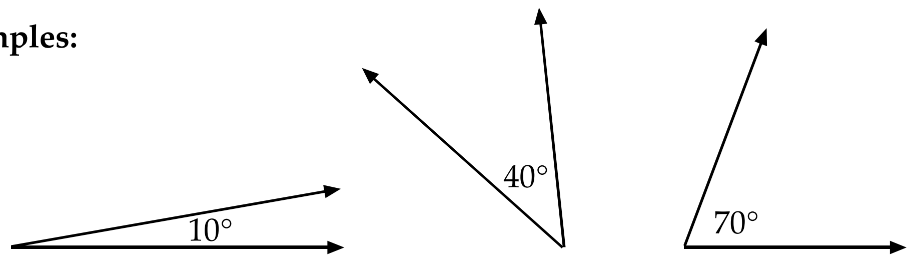
Angles are named for the way they relate to 90 degrees and 180 degrees.

acute angle	=	$< 90°$
right angle	=	$90°$
obtuse angle	=	$> 90°$ and $< 180°$
straight angle	=	$180°$



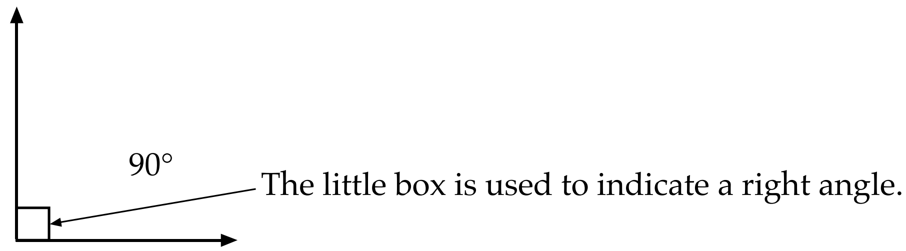
An *acute angle* measures greater than 0° but less than 90°.

Examples:



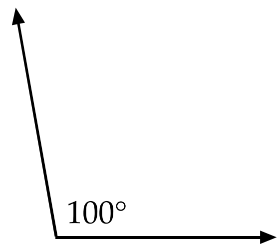
A *right angle* measures exactly 90°.

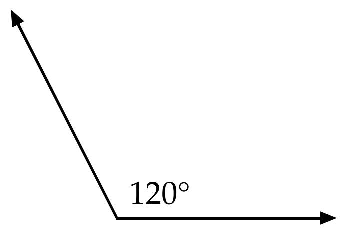
Example:



An *obtuse angle* measures greater than 90° but less than 180°.

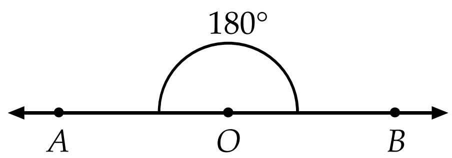
Examples:





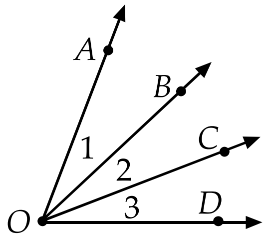
A *straight angle* measures exactly 180°.

Example:



$\angle AOB$ has vertex O and the measure of $\angle AOB$ is 180°. In this case, we need to use three letters to name the angle.

When we name angles, if two or more angles share a ray and have a common vertex, then three letters are used to name each angle.



In the figure above:

The top angle ($\angle AOB$) can also be named $\angle 1$ or $\angle BOA$, but not $\angle O$.

$\angle 2$ can be named $\angle BOC$ or $\angle COB$.

$\angle 3$ can be named $\angle COD$ or $\angle DOC$.

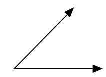
Also, $\angle AOC$ is composed of $\angle 1 + \angle 2$ and

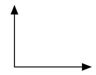
$\angle BOD$ is composed of $\angle 2 + \angle 3$ and

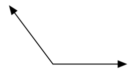
$\angle AOD$ is composed of $\angle 1 + \angle 2 + \angle 3$

Practice

Match each **description** *with the correct term.*

______ 1. < 90° 

______ 2. = 90° 

______ 3. > 90° and < 180° 

______ 4. = 180°

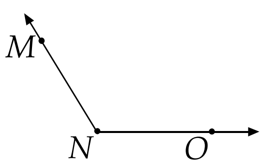
A. acute angle

B. obtuse angle

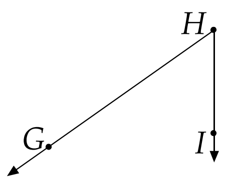
C. right angle

D. straight angle

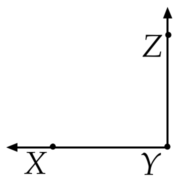
Compare each **angle** *below with a* **right angle**. *Then write whether the angle is* **acute**, **obtuse**, *or* **right**.



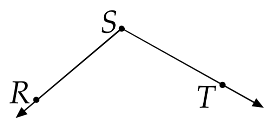
5. ____________ angle



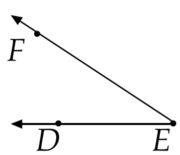
8. ____________ angle



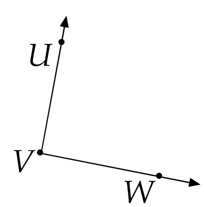
6. ____________ angle



9. ____________ angle



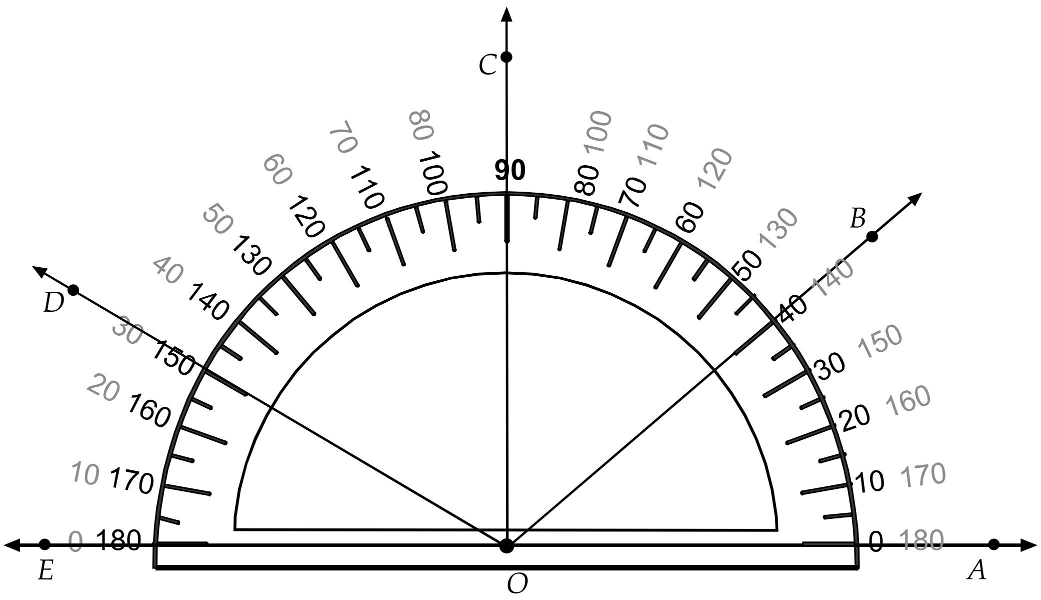
7. ____________ angle



10. ____________ angle

Practice

Use the **protractor** *below to find the* **measure of each angle**. *Then write whether the angle is* **acute**, **right**, *or* **obtuse**.



1. ∠*AOB* ______________________________

2. ∠*AOC* ______________________________

3. ∠*BOD* ______________________________

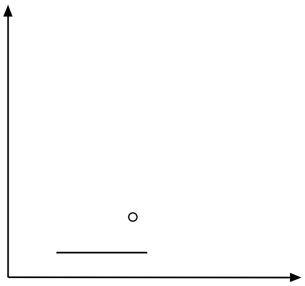
4. ∠*COD* ______________________________

5. ∠*COE* ______________________________

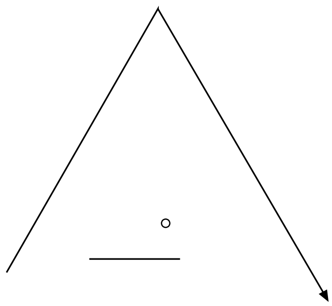
6. ∠*DOE* ______________________________

Use a **protractor** *to* **measure** *each* **angle** *below.*

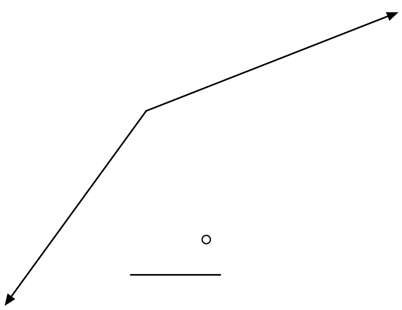
7. ________°



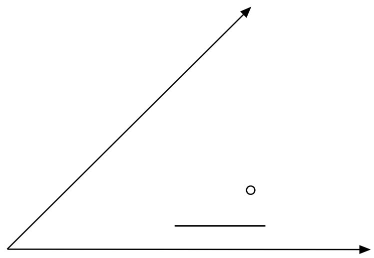
8. ________°



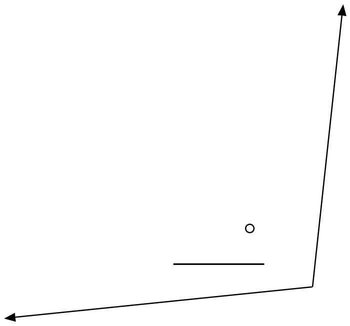
9. ________°



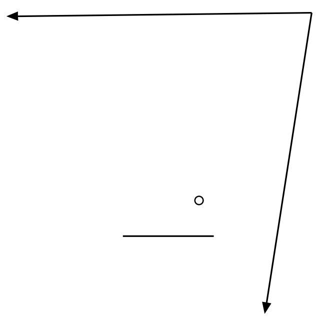
10. ________°



11. ________°



12. ________°



Use a **protractor** *to* **draw angles** *having these* **measures**.

13. 60°

14. 120°

15. 90°

16. 20°

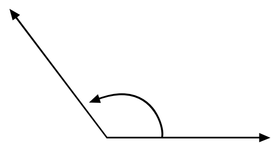
17. 160°

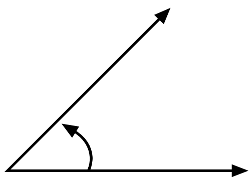
18. 45°

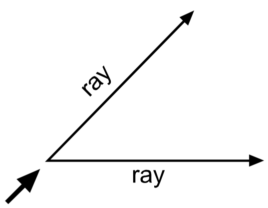
19. 100°

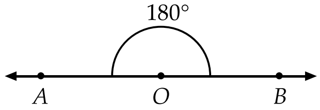
Practice

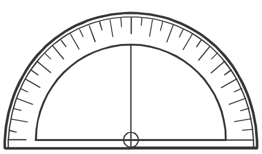
Match each **graphic** *with the correct term. Write the letter on the line provided.*

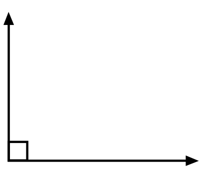
_______ 1. 

_______ 2. 

_______ 3. 

_______ 4. 

_______ 5. 

_______ 6. 

A. acute angle

B. obtuse angle

C. protractor

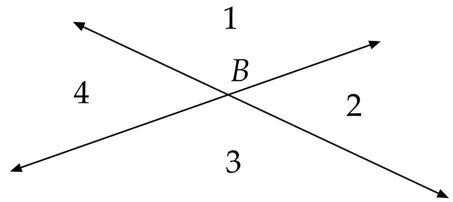
D. right angle

E. straight angle

F. vertex

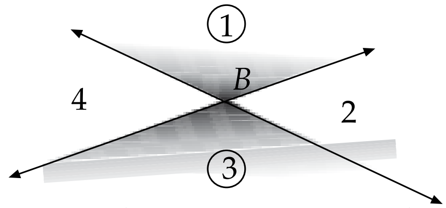
More Special Angles

Suppose we draw 2 lines that intersect:



Notice that 4 angles are formed. Which angles appear to be congruent? Test your theory by measuring each angle.

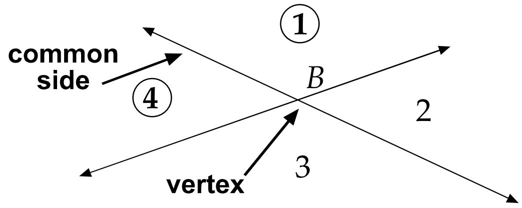
You should have found that angle 1 and angle 3 are congruent and angle 2 and angle 4 are congruent.



Angle 1 and angle 3 are congruent and angle 2 and angle 4 are congruent.

When two lines intersect, angles that are *opposite* or directly across from each other are called **vertical angles**. Vertical angles are always congruent.

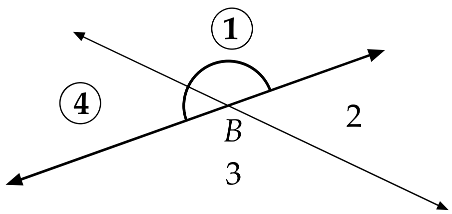
Angle 1 and angle 4 have a common side and also the same vertex. We say that they are **adjacent angles** because they are next to each other.



Angle 1 and angle 4 have a common side and also the same vertex. We say they are adjacent angles because they are next to each other.

Angle 1 and angle 2 are also *adjacent angles*, as well as angle 2 and angle 3, and angle 3 and angle 4.

Again look at angle 1 and angle 4. We see that the two angles together make a straight angle.



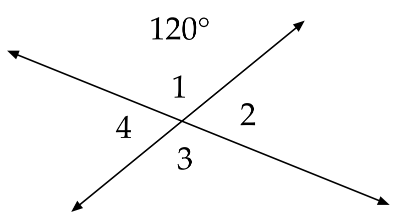
Angle 1 and angle 4 together make a straight angle.

Remember, a straight angle is an angle that measures 180 degrees. These two angles share a special relationship:

Two angles are said to be **supplementary angles** if the **sum** of their measures is 180 degrees. Therefore angle 1 and angle 4 are *supplementary angles*.

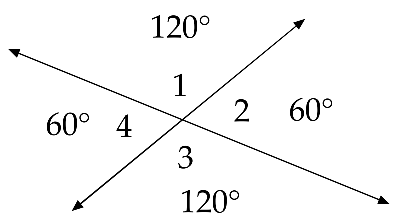
Example: Let's consider the following situation.

Suppose that the measure of angle 1 is 120 degrees. Can we find the measure of the other angles without measuring?



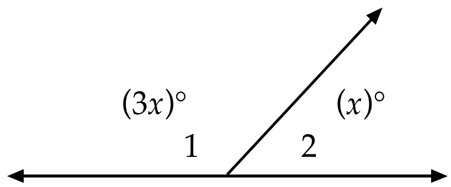
Remember that vertical angles (opposite angles) are congruent. Therefore the measure of angle 3 is also 120 degrees.

Angle 1 and angle 4 are supplementary angles (measure of angles add to equal 180 degrees). If angle 1 is 120 degrees, then angle 4 has to be 60 degrees.

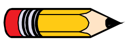


Angle 2 and angle 4 are vertical angles, so if angle 4 is 60 degrees, then angle 2 is also 60 degrees.

Example: Study the following problem.



It appears that angle 1 and angle 2 are supplementary. If the angles are supplementary, their measures add to 180 degrees:

 **Remember:** m∠1 means measure of angle 1

$$\begin{aligned} m\angle 1 + m\angle 2 &= 180^\circ \\ (3x)^\circ + (x)^\circ &= 180^\circ && \text{combine like terms} \\ (4x)^\circ &= 180^\circ \\ \frac{4x^\circ}{4} &= \frac{180^\circ}{4} && \text{divide both sides by 4} \\ x &= 45^\circ \end{aligned}$$

To check, **substitute** 45 for the **variable** *x* in the **equation**.

Measure of angle 1:

$$\begin{aligned} 3x &= 3(45) \\ &= 135 \text{ degrees} \end{aligned}$$

Measure of angle 2:

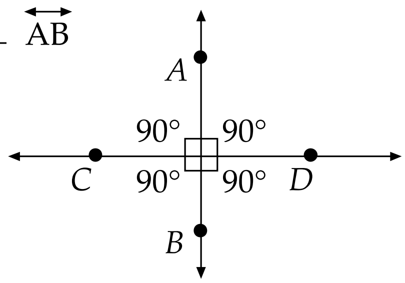
$$x = 45 \text{ degrees}$$

Check: 135 degrees + 45 degrees = 180 degrees *It checks!*

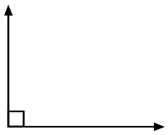
What if we have 2 intersecting lines and all 4 of the angles formed measure 90 degrees? In this situation, we say that the lines are **perpendicular lines**. They form right angles. Two lines (or rays, or segments) that meet at right angles are **perpendicular (⊥)**. *Perpendicular lines* form right angles where they meet.

Note: The symbol ⊥ means *is perpendicular to*.

$\overleftrightarrow{AB} \perp \overleftrightarrow{AB}$



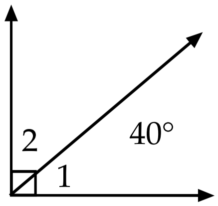
Remember: If you see an angle with a little box "inside it," then you know that the measure of the angle is 90 degrees and that the angle is a right angle.



Our discussion of right angles and perpendicular lines leads to the last definition in our section.

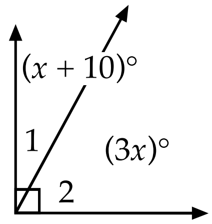
Two angles are **complementary angles** if the sum of their measures is 90 degrees.

Example: Study the following picture.



Angle 1 and angle 2 are *complementary angles* because the sum of their measures is 90 degrees. (Note the little box in the corner). If angle 1 measures 40 degrees, then angle 2 would have to be 50 degrees, because 40 + 50 = 90.

Example: Study the following picture.



This problem is the same as the one before it, only a little bit harder.

Since we see a little box in the corner, we know that the two angles are complementary. The sum of their measures is 90 degrees.

$$
\begin{aligned}
m\angle 1 + m\angle 2 &= 90^\circ \\
(x + 10)^\circ + (3x)^\circ &= 90^\circ \\
(4x) + (10^\circ - 10^\circ) &= 90^\circ - 10^\circ && \text{combine like terms and} \\
4x &= 80^\circ && \text{subtract 10 from both sides} \\
\frac{4x}{4} &= \frac{80}{4} && \text{divide both sides by 4} \\
x &= 20^\circ
\end{aligned}
$$

To check, *substitute* 20 for the variable x.

Measure of angle 1:

$$
\begin{aligned}
x + 10 &= 20 + 10 \\
&= 30 \text{ degrees}
\end{aligned}
$$

Measure of angle 2:

$$
\begin{aligned}
3x &= 3(20) \\
&= 60 \text{ degrees}
\end{aligned}
$$

Check: 30 degrees + 60 degrees = 90 degrees *It checks!*

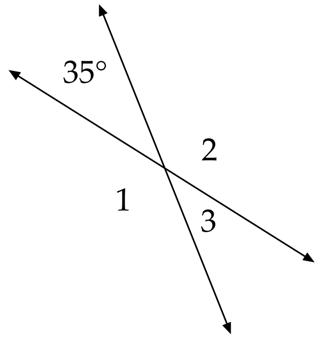
Practice

Find the **measures of these angles**. *Show all your work.*

1. m∠1 = ____________

   m∠2 = ____________

   m∠3 = ____________

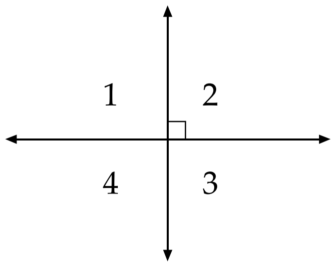


2. m∠1 = ____________

   m∠2 = ____________

   m∠3 = ____________

   m∠4 = ____________



Answer the following. Show all your work.

3. ∠*A* and ∠*B* are complementary. If the m∠*A* is 28 degrees, what is the

   m∠*B*? ____________

4. ∠*A* and ∠*B* are supplementary. If the m∠*A* is 28 degrees, what is the

   m∠*B*? ____________

5. Determine if the angles are complementary *or* supplementary.

   a. m$\angle A$ = 15 degrees and m$\angle B$ = 75 degrees

      The angles are ________________________________ .

   b. m$\angle C$ = 100 degrees and m$\angle D$ = 80 degrees

      The angles are ________________________________ .

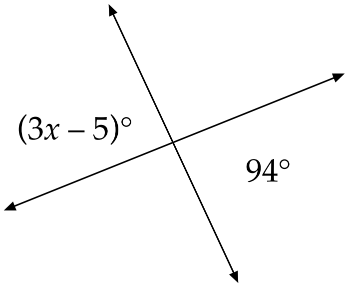
   c. m$\angle E$ = 90 degrees and m$\angle F$ is a right angle

      The angles are ________________________________ .

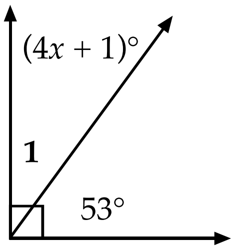
   d. m$\angle G$ = $(2x - 90)$ degrees and m$\angle H$ = $(180 - 2x)$ degrees

      The angles are ________________________________ .

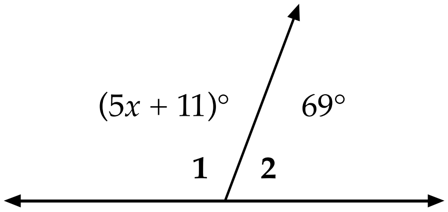
6. Solve for x. ____________



7. Solve for x. Find the measure of angle 1. ______________________



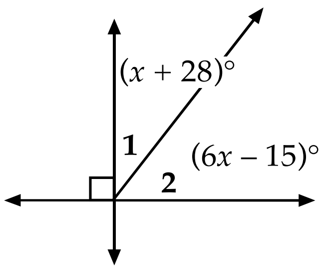
8. Solve for x. Find the measure of angle 1. ______________________



Find the **measures of these angles**. *Show all your work.*

9. m∠1 = ____________

   m∠2 = ____________

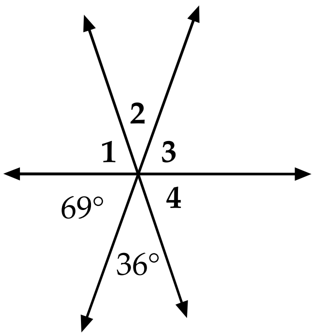


10. m∠1 = ____________

    m∠2 = ____________

    m∠3 = ____________

    m∠4 = ____________

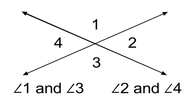


Practice

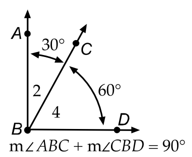
Use the list below to write the correct term for each definition on the line provided.

adjacent angles	**perpendicular lines**	**supplementary angles**
complementary angles	**substitute**	**variable**
degree (°)	**sum**	**vertical angles**
measure (m) of an angle (∠)		

______________________ 1. the opposite angles formed when two lines intersect

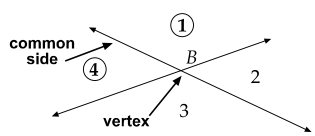


______________________ 2. two angles, the sum of which is exactly 90°

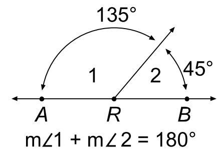


______________________ 3. the result of an addition

______________________ 4. two angles having a common vertex and sharing a common side



______________________ 5. two angles, the sum of which is exactly 180°



______________________ 6. the number of degrees (°) of an angle

______________________ 7. common unit used in measuring angles

______________________ 8. to replace a variable with a numeral

______________________ 9. any symbol that could represent a number

______________________ 10. two lines that intersect to form right angles